



TORONTO INTERNATIONAL OSLIFE SCHOOL
COURSE OUTLINE

Calculus & Vectors, Grade 12, University Preparation

Name of School: Toronto International Oslife School

Department: Mathematics

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Course Development Date: June 2018

Course Reviser: John Chalmers, B.Mus., B.Ed. OCT

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Course Title & Grade: Calculus & Vectors, Grade 12

Ministry Course Code: MCV4U

Course Type: University Preparation

Credit Value: 1.0

Developed from: The Ontario Curriculum, Grades 11 & 12, Mathematics, 2007 – Revised

Prerequisite: Advanced Functions (MHF4U) must be taken prior to or concurrently with Calculus and Vectors (MCV4U)

COURSE DESCRIPTION:

This course builds on students' previous experience with functions and their developing understanding of rates of change. Students will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three-dimensional space; broaden their understanding of rates of change to include the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions; and apply these concepts and skills to the modelling of real-world relationships. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended for students who choose to pursue careers in fields such as science, engineering, economics, and some areas of business, including those students who will be required to take a university-level calculus, linear algebra, or physics course.

CURRICULUM OVERALL EXPECTATIONS:

A. RATE OF CHANGE

- A1. Demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit;
- A2. Graph the derivatives of polynomial, sinusoidal, and exponential functions, and make connections between the numeric, graphical, and algebraic representations of a function and its derivative;
- A3. Verify graphically and algebraically the rules for determining derivatives; apply these rules to determine the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions, and simple combinations of functions; and solve related problems.

B. DERIVATIVES AND THEIR APPLICATIONS

- B1. Make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;
- B2. Solve problems, including optimization problems that require the use of the



concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models.

C. GEOMETRY AND ALGEBRA OF VECTORS

- C1. Demonstrate an understanding of vectors in two-space and three-space by representing them algebraically and geometrically and by recognizing their applications;
- C2. Perform operations on vectors in two-space and three-space, and use the properties of these operations to solve problems, including those arising from real-world applications;
- C3. Distinguish between the geometric representations of a single linear equation or a system of two linear equations in two-space and three-space, and determine different geometric configurations of lines and planes in three-space;
- C4. Represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections.

D. TRIGONOMETRIC FUNCTIONS

- D1. determine the values of the trigonometric ratios for angles less than 360° ; prove simple trigonometric identities; and solve problems using the primary trigonometric ratios, the sine law, and the cosine law;
- D2. demonstrate an understanding of periodic relationships and sinusoidal functions, and make connections between the numeric, graphical, and algebraic representations of sinusoidal functions;
- D3. identify and represent sinusoidal functions, and solve problems involving sinusoidal functions, including problems arising from real-world applications.

COURSE CONTENT:

Unit Name	Time Allocation
1. Introduction to Calculus - <i>This unit continues on from the Rates of Change unit in Advanced Functions.</i>	13.5
2. Derivatives - <i>This unit will introduce student to shortcuts to taking derivatives, such as the product, quotient and chain rules</i>	12.5
3. Derivatives and their Applications - <i>second derivatives will be discussed, as well as the real world and graphical applications of derivatives.</i>	14.5
4. Increasing and Decreasing Functions - <i>The concepts of instantaneous rate of change, tangent lines, and first principles will be discussed.</i>	12.5
5. The Derivatives of Exponential Functions - <i>The concept of the derivative function will be worked on and how first principles can be used to develop an equation that models the slope of the tangent.</i>	9.5
6. Introduction to Vectors - <i>Students will be introduced to vectors and various applications, such as the dot and cross product. Various forms of equations of lines and planes will be introduced to the student, and the advantages/disadvantages of each will be discussed.</i>	13.5
7. Vectors as Force Velocity - <i>Students will learn how to determine the nature of intersection between lines and planes in 3D.</i>	13.5
8. Equations in R^2 and R^3 - <i>Students will apply concepts from the previous unit to determine angles between planes, orthogonal</i>	18.5



<i>solutions, and learn to solve and sketch systems of equations</i>	
9. Final Exam (30% of final mark)	2
Total Hours	110

TEACHING & LEARNING STRATEGIES:

Direct Instruction (teacher-led)	✓	Class Activity (teacher facilitation)	
Direct Instruction (discussion possible)		Experiential learning (learn by doing)	
Class Discussion (teacher facilitated)	✓	Worksheets/Surveys	✓
Small Group Discussion		Individual or Group Research	✓
Partner Discussion/Conferencing		Teacher modeling	
1:1 Conferencing Teacher & Student	✓	Text-based modeling	
Teacher reading to class		Use of Computers / Internet	✓
Silent individual reading	✓	Use of video or audio materials	✓
Group based reading		Role Playing	
Independent Work (teacher facilitation)	✓	Presentations	✓
Group Work (teacher facilitation)		Guest Speaker / Interviews / Questions	
Brainstorming		Field Trip	

ASSESSMENT & EVALUATION:

Purpose: The primary purpose of assessment is to improve student learning. Assessment relates directly to the expectations for the course.

A variety of assessments for and as learning are conducted on a regular basis to allow ample opportunities for students to improve and ultimately demonstrate their full range of learning and in order for the teacher to gather information to provide feedback. Assessment tasks relate to the success criteria set out in lesson plans. Success criteria allow students to see what quality looks like.

Evaluation is the process of judging the quality of student work in relation to the achievement chart categories and criteria, and assigning a percentage grade to represent that quality. Evaluation is based on gathering evidence of student achievement through:

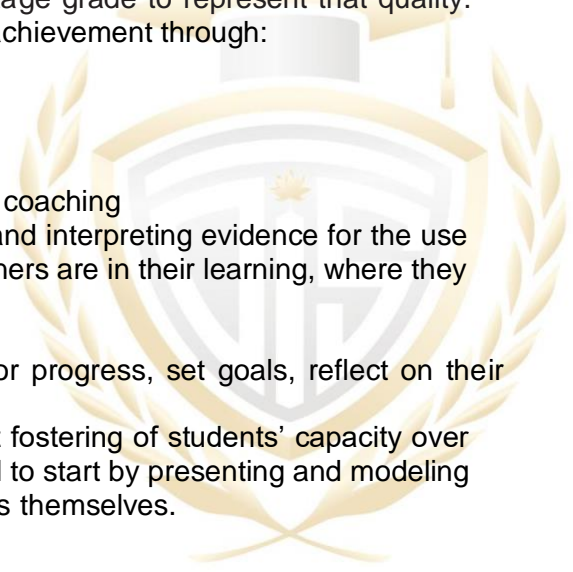
- Products
- Observations
- Conversations

Assessment for Learning - we provide feedback and coaching

Assessment FOR Learning is the process of seeking and interpreting evidence for the use of learners and their teachers to decide where the learners are in their learning, where they need to go, and how best to go there.

Assessment as Learning - we help students monitor progress, set goals, reflect on their learning

Assessment AS Learning is the process of the explicit fostering of students' capacity over time to be their own best assessors, but teachers need to start by presenting and modeling external, structured opportunities for students to assess themselves.





Assessment of Learning – we use assessments as ways of providing evaluative statements about the level of achievement of students

Assessment OF Learning is the assessment that becomes public and results in statements of symbols (marks/grades/levels of achievement) about how well students are learning. It often contributes to pivotal decisions that will affect students' future.

Grading

- The final grade is based on performance in 3 areas: products, observations and conversations.
- 70% of the grade is based on evaluations conducted throughout the course.
- 30% is based on a final evaluation.

Weighting of Categories:

Knowledge & Understanding	Thinking	Communication	Application
30%	20%	20%	30%

Assessment Tools: The following assessment tools are used in MCC4U at TIOS:

Marking schemes	✓	Rubrics	✓
Anecdotal comments		Checklists	✓
Rating Scales			

Assessment Strategies: The following assessment strategies are used in MCV4UU at TIOS:

Assessment for Learning		Assessment as Learning		Assessment of Learning	
Quizzes	✓	Journal	✓	Tests	✓
Tests		Exit and Entrance Cards	✓	Presentations	✓
Presentations		KWL Chart	✓	Journals	
Journals	✓	Self/Peer assessment	✓	Essays	
Essays		Logs		Models	
Models				Projects	✓
Projects				Demonstrations	
Demonstrations				Conferencing	
Conferencing	✓			Questioning	
Questioning				Independent Study Assignment	
Independent Study Assignment				Art Exhibits	
Art Exhibits				Researching	
Researching	✓			Reading Aloud	
Reading Aloud				Problem Solving	✓



				(process focused)	
Problem Solving (process focused)	✓			Debates	
Debates	✓			Work Sheets	
Work Sheets				Role Playing	
Role Playing				Direct Instruction	
Direct Instruction					

CONSIDERATIONS FOR PROGRAM PLANNING:

Instructional Approaches

Teachers at TIOS are expected to:

- clarify the purpose for learning;
- help students activate prior knowledge;
- differentiate instruction for individual students and small groups according to need;
- explicitly teach and model learning strategies;
- encourage students to talk through their thinking and learning processes;
- provide many opportunities for students to practise and apply their developing knowledge and skills;
- apply effective teaching approaches involve students in the use of higher-level thinking skills;
- encourage students to look beyond the literal meaning of texts and artistic works;
- encourage students to rehearse, practice, apply, skills and strategies, and to make their own choices.

Teachers use a variety of instructional and learning strategies best suited to the particular type of learning. Students have opportunities to learn in a variety of ways:

- individually;
- cooperatively;
- independently with teacher direction;
- through investigation involving hands-on experience;
- through examples followed by practice;
- by using concrete learning tools;
- by encouraging students to gain experience with varied and interesting applications of the new knowledge. Rich contexts for learning open the door for students to see the “big ideas” of mathematics that will enable and encourage them;
- to reason mathematically throughout their lives.

Promoting Positive Attitudes Towards Learning Mathematics

Teachers must be careful to build a positive environment in which students may study mathematics; students that enjoy the courses are more likely to do well and enroll in more advanced mathematics courses.

Teachers can set students up for developing positive attitudes by providing opportunities for them to:

- be engaged in making mathematical conjectures



- experience breakthroughs as they solve problems
- see connections between important ideas
- see their teacher's enthusiasm about teaching mathematics

Teachers must be mindful students developing negative attitudes whether through a feeling of inadequacy or anxiety from not solving problems quickly, easily, or in the correct manner. Students should be able to recognize that:

- There are many correct ways to come to a solution
- Problem solving requires time and effort to learn, and requires perseverance
- With this perseverance comes the ability to move past barriers and overcome the frustration of getting stuck

Teachers can encourage students to keep trying a problem when they are stuck and guide them through routes of thought to arrive at a solution, as well as encourage students to develop this perseverance as being challenged and overcoming barriers are cornerstones of education.

Teachers must be mindful of their students' confidence in their skills in order for them to continue seeing success and understanding in their studies.

Program Considerations for Students with Special Education Needs

Teachers must incorporate appropriate strategies for instruction and assessment to facilitate the success of students with special education needs in their classrooms. These strategies stem from the beliefs as laid out in *Special Education Transformation: The report of the Co-Chairs with the Recommendations of the Working Table on Special Education, 2006*:

- All students can succeed.
- Universal design and differentiated instruction are effective and interconnected means of meeting the learning or productivity needs of any group of students.
- Successful instructional practices are founded on evidence-based research, tempered by experience.
- Classroom teachers are key educators for a student's literacy and numeracy development.
- Each student has his or her own unique patterns of learning.
- Classroom teachers need the support of the larger community to create a learning environment that supports students with special education needs.
- Fairness is not sameness.

Teachers must plan their program that recognize the diversity of students' learning styles, needs, and responses, so students can have performance tasks that respect their abilities so they can derive the greatest possible benefit from the teaching and learning process.

Teachers must be mindful of three types of accommodations for students:

- Instructional Accommodations: changes in teaching strategies, including styles of presentation, methods of organization, or use of technology and multimedia
- Environmental Accommodations: changes that the student may require in the classroom and/or school environment, such as preferential seating or special lighting.
- Assessment accommodations: changes in assessment procedures that enable the student to demonstrate his or her learning, such as allowing additional time to complete tests or assignments, or permitting oral responses to test questions



No modifications to course expectations are made at this school.

Program Considerations for English Language Learners

TIOS Teachers will use appropriate strategies for instruction and assessment to facilitate the success of the English language learners in their classrooms. These strategies include (but are not limited to):

- modification of some or all of the subject expectations depending on the level of English proficiency;
- use of a variety of instructional strategies (e.g., extensive use of visual cues, graphic organizers, scaffolding);
- previewing of textbooks;
- pre-teaching of key vocabulary;
- peer tutoring;
- strategic use of students' first language;
- use of a variety of learning resources (e.g., visual material, simplified text, bilingual dictionaries and materials that reflect cultural diversity);
- use of assessment accommodations (e.g., granting of extra time); and
- use of oral interviews, demonstrations or visual representations or tasks requiring completion of graphic organizers and cloze sentences instead of essay questions and other assessment tasks that depend heavily on proficiency in English.

Environmental Education in the Mathematics Classroom

Acting Today, Shaping Tomorrow: A Policy Framework for Environmental Education in Ontario Schools outlines an approach to environmental education that recognizes the needs of all Ontario Students and promotes environmental responsibility in the operations of all levels of the education system.

The goals outlined are surrounded around: teaching and learning, student engagement and community connections, and environmental leadership. By promoting the study of issues and solutions, the engagement of practicing and promoting environmental leadership and stewardship, and the importance of leadership through responsible promotion of environmentally safe and secure practices.

Equity & Inclusion

At Toronto International OSLife School, our mission is to inspire academic excellence for students and strong confidence for parents.

The following statements of belief are excerpted from the Peel District School Board document entitled *Empowering Modern Learners (Addendum 2020)* and help us to shape a foundation that determines our school's belief system about our students.

As a school, we are committed to ensuring that each student is represented and reflected in the learning experiences and learning environments we provide. This means that teachers and administrators at TIOS celebrate and value unique student interests, backgrounds, cultures and prior experiences. Our beliefs as a staff are founded on the following belief system that we bring to classrooms every day:



- We must actively confront inequities and barriers that uphold racism and other forms of oppression so learners of all identities are empowered through education that embraces their identities and lived experiences.
- Each learner is curious, competent and able to take an active role in his or her own learning.
- Effective educators empower all learners to achieve personal excellence by being open, flexible and responsive to their needs.
- A positive, innovative learning environment empowers all of us to grow through rich, authentic relationships both locally and globally.
- Together as a community of families, educators and leaders, we share responsibility to inspire our modern learners to be active, critically engaged, global citizens.

Culturally responsive pedagogy and modern digital tools offer unprecedented opportunities to empower historically under-served learners by providing barrier-free access to information and learning networks. This access generates new opportunities for learners to explore their passions, share their voices and consider diverse perspective.

Being an educator that embodies Modern Learning is more about the journey than the destination. As we acknowledge our past and commit to a new future, we will focus on instructional strategies that take in consideration student learning needs, and assessment practices that are equitable, transparent and focus on student achievement. Our classroom must foster critical thought, adaptability and innovation and understanding that these concepts look different and have different implications for each learner.

Literacy, Mathematical Literacy, and Inquiry/Research Skills

TIOS emphasizes the importance of the following:

- using clear, concise communication in the classroom involving the use of diagrams, charts, tables, and graphs
- emphasizing students' ability to interpret and use graphic texts.
- acquiring the skills to locate relevant information from a variety of sources, such as books, newspapers, dictionaries, encyclopedias, interviews, videos, and the Internet.
- learning that all sources of information have a particular point of view
- learning that the recipient of the information has a responsibility to evaluate it, determine its validity and relevance, and use it in appropriate ways.

The Role of Technology

Information and communications technologies (ICT) tools used in many ways:

- Students use multimedia resources, databases, Internet websites, digital cameras, and word-processing programs.
- Students use databases, spreadsheets, dynamic geometry and statistical software, graphing software, computer algebra systems, and so on in order to quickly navigate through complex problems, to see the effect of dynamic data on their values and trends, and to see a graphical representation of data.
- They use technology to collect, organize, and sort the data they gather and to write, edit, and present reports on their findings.
- Students are encouraged to use ICT to support and communicate their learning. For example, students working individually or in groups can use computer technology and/or Internet websites to gain access to museums and archives in Canada and around the world.
- Students use digital cameras and projectors to design and present the results of



their research to their classmates.

- The school plans to use ICT to connect students to other schools and to bring the global community into the classroom.
- Students are made aware of issues of Internet privacy, safety, and responsible use, as well as of the potential for abuse of this technology, particularly when it is used to promote hatred.

Career Education

Students are given opportunities to develop career-related skills by:

- applying their skills to work-related situations;
- exploring educational and career options;
- developing research skills;
- developing key essential skills such as reading text, writing, computer use, measurement and calculation, and problem solving;
- practising expository writing;
- learning strategies for understanding informational reading material;
- making oral presentations;
- working in small groups with classmates to help students express themselves confidently and work cooperatively with others.

Health and Safety in Mathematics

In Mathematics courses, students must be familiar with the concept that health and safety is everyone's responsibility at all times, not just in the school. While these courses are, for the most part, very academic, there are still many opportunities to practice health and safety strategies.

While there is no direct mention of these practices throughout the curriculum for these courses, students can use the context of the assessment to build awareness and understanding of health and safety issues.

Field trips, while very beneficial to students educational experiences, are also opportunities for appropriate health and safety strategies to be planned and employed as there can be many unpredictable settings and events that take place outside of the classroom.

Financial Literacy

The school is emphasizing the importance of ensuring that Ontario students have the opportunity to improve their financial literacy. Financial literacy is defined as "having the knowledge and skills needed to make responsible economic and financial decisions with competence and confidence". The goal is to help students acquire the knowledge and skills that will enable them to understand and respond to complex issues regarding their own personal finances and the finances of their families, as well as to develop an understanding of local and global effects of world economic forces and the social, environmental, and ethical implications of their own choices as consumers. Thus, an attempt will be made to integrate Financial Literacy in all the school's courses.

Academic Honesty

Plagiarism occurs when someone presents the work of others as their own. This would include copying large amounts of text from the Internet or other written texts without crediting



the original author. Plagiarism also occurs when someone copies the work of other students, pretending it to be their own. Surprisingly, the third instance of plagiarism occurs when a student copies an assignment from one course for use in a different course, pretending it to be original work. All three of these instances constitute plagiarism and are very serious breaches of academic honesty.

In many post-secondary institutions, students who are found to be guilty of academic dishonesty (plagiarism) are forced to withdraw from the course and/or the university.

At TIOS, we consider the issue of academic honesty to be very important. Since our goal is to inspire academic excellence for students, we believe that academic honesty is an important lesson to learn during high school. For this reason, teachers in each course will review the correct methods of footnoting sources so that students can avoid any suspicion of copying from outside sources. Using someone else's ideas to support your own is not the crime – but when you pretend that someone else's ideas are yours, that becomes the problem! Teachers throughout Ontario have access to online software that easily detects plagiarism, so it is important for students to pay careful attention to this issue.

As outlined in *Growing Success*, students must understand that the tests/exams they complete and the assignments they submit for evaluation must be their own work and that cheating and plagiarism will not be condoned.

- **Responsibilities of TIOS Students:** When the teacher asks you to use your own words and ideas, it means that you should use your own words and your own ideas. You must demonstrate to the teacher that you are capable of submitting work that is your own. When a teacher asks to put your ideas into your own words, it does not mean that there is a correct answer for the assignment. It means that you have to come up with your own ideas to give to the teacher.
- **Responsibilities of TIOS Teachers:** Teachers will help students avoid plagiarising by using some of the following strategies:
 - defining the term “plagiarism” and reminding students of the policy when setting out an assignment;
 - giving students examples of what plagiarism looks like;
 - emphasizing the importance of using process skills to arrive at a product;
 - teaching students research skills so they can avoid plagiarising: note taking, paraphrasing, summarizing;
 - teaching students organizational skills: finding and organizing information to build understanding of a topic;
 - teaching students how to make an outline for a report or research essay;
 - having students keep a learning log to reflect on what they learned through the process: how research and organizational skills helped with the project, how could the product be improved, how can the research and organizational skills be improved;
 - assessing the process steps: notes, outline, summary, bibliography, drafts, etc.; and/or
 - informing students of the consequences of plagiarism.

Consequences for Academic Dishonesty at TIOS

Students found to have plagiarized assignments at TIOS will be subject to a series of escalating consequences:



- Instance #1: When plagiarism has been detected, the teacher will discuss the matter with the student. Both parents and the Principal will be informed of the details and the student will have the opportunity of redoing the assignment in a way that avoids plagiarism.
- Instance #2: When plagiarism is detected a second time, the student will receive a mark of zero for the assignment. Parents and the Principal will once again be informed and the Principal will note this in their school records.
- Instance #3: Repeat instances of plagiarism may result in withdrawal from the course and/or the school without refund of tuition. Similarly, students who are guilty of cheating on tests or examinations will receive a mark of zero on the test or examination and these details will also be noted in their school records.

Appeal

- A student may appeal the teacher's decision to the Principal after discussion with the teacher.

Late Assignments

Students are responsible for providing evidence of their achievement of the overall expectations within the time frame specified by the teacher and in a format approved by the teacher. There are consequences for not completing assignments for evaluation or for submitting those assignments late.

Core Resources

- Calculus and Vectors, Nelson (2008)
- Calculus and Vectors 12, McGraw Hill Ryerson (2008)
- Moodle website with video link

Supplemental Resources

- Elementary Linear Algebra Third Edition, Howard Anton (1981)
- Mathematics Manual, Frederick S. Merritt (1962)

Various Online Resources including:

- La Citadelle, Iulia & Teodru Gugoiu - <https://www.la-citadelle.com/courses/calculus/>
- MCV4U - Calculus and Vectors, TheDovoin - <https://youtube.com/playlist?list=PLv4HmJOvGRVyrWOkSaLTWRGeYk-K9wJZn>
- Khan Academy - <https://www.khanacademy.org/math/differential-calculus>
- Math is Fun Advanced: Calculus - <https://www.mathsisfun.com/calculus/>
- Paul's Online Notes - <https://tutorial.math.lamar.edu/Classes/Calcl/Calcl.aspx>